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GLOBAL CONTROL OF VIBRATION USING A TUNABLE VIBRATION NEUTRALIZER

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Tunable vibration neutralizers are currently used to control harmonic vibration of structures at the point at which they are attached, and the way in which the characteristics of the neutralizers influence their effectiveness in this role are reasonably well understood. However, the use of tunable vibration neutralizers to control global vibration of a structure has not been so well explored, and the aim of this paper is to investigate the parameters of the neutralizer that influence the control of the vibrational kinetic energy of a structure. It is shown that in general a de-tuned rather than a tuned vibration neutralizer is required at most frequencies, offering a stiffness-like or a mass-like impedance to the structure. However, because this control strategy requires a measure of the global vibration of the host structure, which can be difficult and/or costly in practice, it is preferable to use a tuned vibration neutralizer. For this to be effective it is demonstrated that the tuned neutralizer needs to be appropriately positioned on the structure. If this is the case then such a device can be almost as effective as a de-tuned neutralizer over quite a wide range of frequencies. An expression to determine the optimum ratio of the neutralizer mass to the modal mass of the structure is also derived.

1. INTRODUCTION

Since the vibration absorber was described by Ormodroyd and den Hartog in 1928 [1], it has been used in many applications [2]. Although many researchers and engineers still call it the vibration absorber, it has been called a vibration neutralizer by others, for example references [2, 3], and it is referred to by this name in this paper. A fairly recent development has been to make the device adaptive, by making the stiffness adjustable, so that it can track changes in the excitation frequency [4–7], and ways of achieving this have been reviewed by Von Flotow *et al.* [5] and Brennan [8]. One application of this device that is worthy of mention is the control of transmitted vibration from the engine of a DC9 aircraft to the fuselage and hence a reduction in sound pressure inside the cabin [9]. These types of systems, called adaptive–passive by Franchek *et al.* [10], are an attractive alternative to passive vibration control. They offer an improved performance over passive measures, and provide a solution, which although does not match that of active control, can potentially work with a very simple control system, such as that described by Long *et al.* [11]. Additionally, because the devices are used at resonance, large forces can be generated, and the only power required is to tune the system, which can be very small. This has obvious benefits with large structures [12]. Buhr *et al.* [13] investigated the use of a tuned neutralizer to control vibration at a point other than the attachment point (non-co-located control), but did not look at the effect of position on the global vibration of a structure.

More recently, researchers have turned their attention to the use of vibration neutralizers to control sound transmission through structures, for example references [14, 15], with the particular application of controlling sound transmission into aircraft in mind. It was found, in this case, that it was preferable to use de-tuned vibration neutralizers rather than resonant devices to reduce the global sound pressure level inside the cabin [16]. Charette *et al.* [17] showed in an experimental investigation that detuned vibration neutralizers were effective in reducing sound radiation from a panel over a frequency range. To achieve this in practice requires a measure of global vibration, which can require a number of sensors to measure the global vibration or sound pressure, and then the use of vibration neutralizers becomes less attractive. The aims of this paper are twofold (i) to investigate, using a simple structure, the conditions under which a tuned vibration neutralizer would be appropriate to control the global vibration of the structure, and (ii) to determine the optimum parameters of a tuned neutralizer used for global vibration control. If the right conditions can be arranged so that a tuned neutralizer rather than a detuned neutralizer can be employed then a simple control strategy can be used [11], and this makes the use of such a resonant device an attractive alternative.

The paper is organized as follows. Following the introduction, section 2 describes a mathematical model, in the modal domain, of the structure and the neutralizers in terms of receptances and dynamic stiffnesses. To gain physical insight into the control mechanisms a simplified model is developed in section 3 that uses only a single tunable vibration neutralizer. This model is used to investigate the effect of the position of the device on the controllability of the host structure as a function of frequency. It is shown that the global vibration of the structure reduces as the tuned dynamic stiffness of the neutralizer increases, but there is a threshold value beyond which no further reductions can be achieved. A general expression to calculate this threshold value is derived. In section 4, a technique used to calculate the optimum secondary force in a fully active control system, is employed to calculate the optimum dynamic stiffness of a vibration control device that minimizes the kinetic energy of the structure. Using this technique, it is clear that a de-tuned neutralizer is required to achieve the minimum global vibration of the structure. The effect of using a tuned, rather than a de-tuned vibration neutralizer is compared with the optimum control strategy and the frequency range when this is acceptable is identified. Finally, the paper is closed with some conclusions in section 5. There is also Appendix A to this paper which describes the matrix algebra required to simplify the mathematical model of a structure with a single neutralizer attached.

2. MATHEMATICAL MODEL OF A STRUCTURE WITH NEUTRALIZERS ATTACHED

Consider a structure that can be modelled using M structural modes and has N neutralizers attached, such as the one shown in Figure 1. The structure and the neutralizers can be modelled separately and then coupled together using the receptance-dynamic stiffness approach [2]. The displacement of the structure w at position x can be written as

$$w(x) = \sum_{m=1}^{M} \phi_m(x) q_m, \tag{1}$$

where $e^{j\omega t}$ time dependence is assumed but is not shown for clarity. $\phi_m(x)$ is the mode shape of the mth mode and the mth modal amplitude is given by $q_m = A_m f_m$, where

$$A_m = \frac{1}{m_m\,(\omega_m^2 - \omega^2 + j2\zeta_m\omega\,\omega_m)} \tag{2}$$

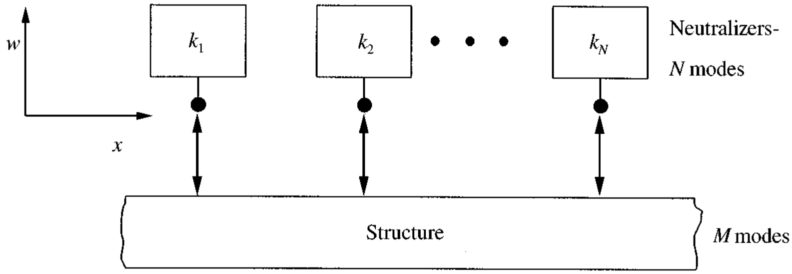

Figure 1. N Neutralizers of dynamic stiffness k_n attached to a structure which has M modes.

and f_m is the generalized force of the mth mode. m_m is the modal mass, ζ_m is the damping ratio and ω_m is the circular natural frequency of the mth mode, respectively. The generalized force of this mode has two contributions, a primary uncontrolled force g_m and the forces applied by the N neutralizers. Thus, the mth modal amplitude is given by

$$q_m = A_m\left(g_m + \sum_{n=1}^{N} f_n \phi(x_n)\right), \tag{3}$$

where x_n denotes the position on the structure at the nth neutralizer and f_n is the force applied to the structure by the nth neutralizer. The vector of modal amplitudes can thus be written as

$$\mathbf{q} = \mathbf{A}[\mathbf{g} + \mathbf{\Phi f}], \tag{4}$$

where $\mathbf{q}$ and $\mathbf{g}$ are the M-length modal amplitude and generalized primary force vectors, respectively, $\mathbf{A}$ is an $(M \times M)$ diagonal matrix whose elements are given by equation (2), $\mathbf{\Phi}$ is the $(M \times N)$ matrix of modal amplitudes where the entry ϕ_{mn} is the modal amplitude of the mth mode evaluated at the nth neutralizer position, and $\mathbf{f}$ is the N-length vector of forces applied to the structure by the neutralizers.

The force applied to the structure by the nth neutralizer can be written in terms of the dynamic stiffness of the neutralizer and the displacement of the structure at the neutralizer position as

$$f_n = -k_n w(x_n), \tag{5}$$

where k_n is the dynamic stiffness of the nth neutralizer and is given by [11]

$$k_n = \frac{-\omega^2 m_n(1 + \mathrm{j}2\zeta_n(\omega/\omega_n))}{1 - (\omega^2/\omega_n^2) + \mathrm{j}2\zeta_n(\omega/\omega_n)}, \tag{6}$$

with m_n is the mass, ζ_n is the damping ratio and ω_n is the circular natural frequency of the nth neutralizer respectively. Substituting for $w(x)$ from equation (1) into equation (5) gives the nth neutralizer force in terms of the modal amplitudes of the structure:

$$f_n = -k_n \sum_{m=1}^{M} \phi_m(x_n) q_m. \tag{7}$$

Thus, the vector of neutralizer forces applied to the structure is given by

$$\mathbf{f} = -\mathbf{K\Phi}^{\mathrm{T}}\mathbf{q} \tag{8}$$

where $\mathbf{K}$ is the $(N \times M)$ diagonal matrix of neutralizer dynamic stiffnesses whose elements are given by equation (6), and T denotes the transpose. Combining equations (4) and (8) gives the coupled matrix equation for the complete system:

$$\mathbf{q} = [\mathbf{I} + \mathbf{A}\boldsymbol{\Phi}\mathbf{K}\boldsymbol{\Phi}^{\mathrm{T}}]^{-1}\mathbf{A}\mathbf{g}, \tag{9}$$

where $\mathbf{I}$ is the identity matrix. The time-averaged kinetic energy E, of the structure is taken as the measure of global vibration of the structure and is proportional to the sum of the squares of the modal velocity amplitudes. For a beam it is given by [18]

$$E = \frac{m\omega^2}{4}\mathbf{q}^{\mathrm{H}}\mathbf{q}, \tag{10}$$

where m is the mass of the beam and the superscript H denotes the Hermitian transpose.

3. GLOBAL CONTROL USING A SINGLE TUNABLE VIBRATION NEUTRALIZER

Considering only one neutralizer attached to the structure, equation (9) becomes

$$\mathbf{q} = [\mathbf{I} + k_n\mathbf{A}\boldsymbol{\phi}(x_n)\boldsymbol{\phi}^{\mathrm{T}}(x_n)]^{-1}\mathbf{A}\mathbf{g}, \tag{11}$$

where k_n is the dynamic stiffness of a single neutralizer given by equation (6) and $\boldsymbol{\phi}(x_n)$ is the vector of mode shapes evaluated at the neutralizer position. This equation can be written in a simplified form, without an inverted matrix, using the procedure detailed in Appendix A as

$$\mathbf{q} = \left[\mathbf{I} - \frac{k_n\mathbf{A}\boldsymbol{\phi}(x_n)\boldsymbol{\phi}^{\mathrm{T}}(x_n)}{1 + k_n\alpha_{nn}}\right]\mathbf{A}\mathbf{g}, \tag{12}$$

where α_{nn} is the point receptance of the structure at the point where the neutralizer is attached and is given by

$$\alpha_{nn} = \sum_{m=1}^{M} A_m\phi_m^2(x_n). \tag{13}$$

If the uncontrolled primary excitation is a single-point force applied at position x_f, equation (12) can be further simplified to

$$\mathbf{q} = \mathbf{A}\left[\boldsymbol{\phi}(x_f) - \boldsymbol{\phi}(x_n)\left(\frac{k_n\alpha_{nf}}{1 + k_n\alpha_{nn}}\right)\right]f, \tag{14}$$

where α_{nf} is the transfer receptance between the primary force and the neutralizer position and $\boldsymbol{\phi}(x_f)$ is the vector of mode shapes evaluated at the primary force position. To examine the differences between global and local control using a single neutralizer the cantilever beam shown in Figure 2(a) is used as an example structure, where m, k and c are the mass, stiffness and damping coefficient of the neutralizer respectively. Because the vibration neutralizer is tunable, the spring stiffness can be adjusted as shown in the figure. If an

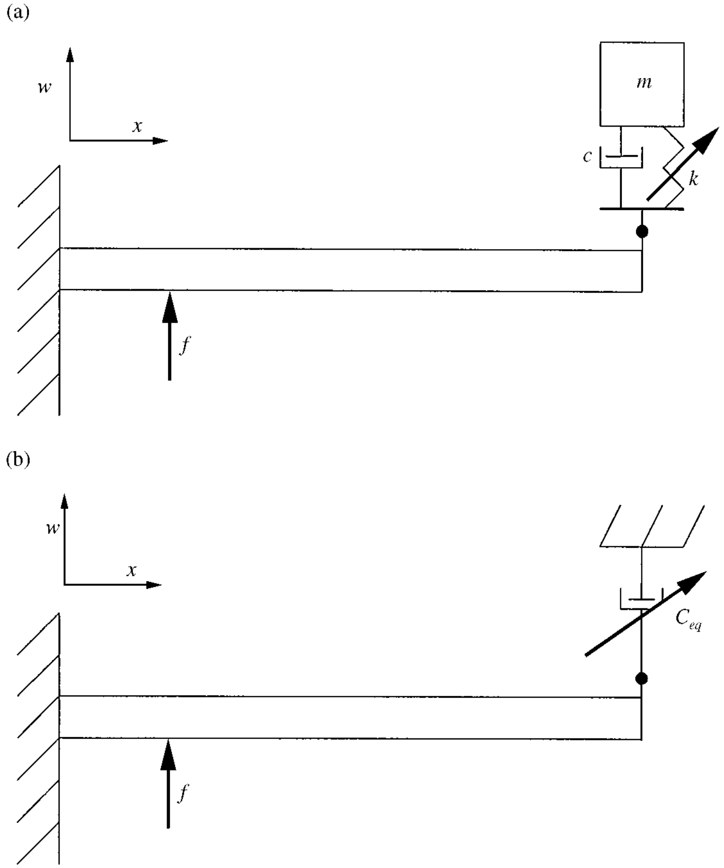

Figure 2. Cantilever beam of length L with (a) a tunable neutralizer attached at the free end, and (b) an equivalent variable viscous damper attached.

aluminium cantilever beam of dimensions 0·5 m × 5 mm × 55 mm, is excited by a point force at $x/L = 0{\cdot}2$ and the neutralizer is tuned to a fixed frequency of 125 Hz, then the kinetic energy of the beam with and without the neutralizer attached is shown in Figure 3. The characteristic dip (marked A) in the kinetic energy can be seen at 125 Hz with a peak on either side of the dip. Calculating the dip in the kinetic energy of the beam *at each frequency*, with the neutralizer tuned so that it is *resonant at each single frequency*, then the resulting plot is shown in Figure 4, with the kinetic energy of the beam alone for comparison. It can be seen from this graph that although there is a reduction in the kinetic energy at some frequencies, at other frequencies the tuned neutralizer actually causes an increase in global vibration. This has been observed by Fuller *et al.* [16] in the control of sound transmission through structures, which is also a global control problem. The reason for this can be seen by examining the tuned dynamic stiffness of the neutralizer. Setting $\omega = \omega_n$ and assuming that damping is small, i.e., $\zeta_n \ll 1$, then from equation (6), the dynamic stiffness of the tuned frequency is given by

$$k_{n\,(tuned)} = \frac{\mathrm{j}\omega^2 m_n}{2\zeta_n}. \tag{15}$$

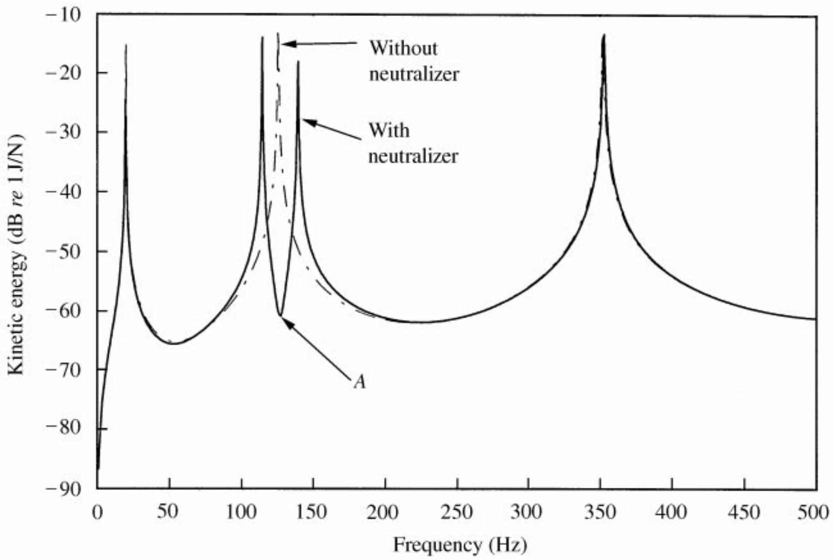

Figure 3. Kinetic energy of a cantilever beam with a neutralizer tuned to 125 Hz and attached to the free end of the beam. Point A denotes the kinetic energy of the beam at the tuned frequency of the neutralizer.

At each frequency, this is equivalent to attaching a damper, which is grounded at one end, to the beam as shown in Figure 2(b). The equivalent damping coefficient is given by

$$c_{eq} = \frac{\omega m_n}{2\zeta_n}. \tag{16}$$

It can be seen that this equivalent damping coefficient increases with mass and frequency and decreases with damping ratio. At frequencies when the effect of the tuned vibration neutralizer is most detrimental (about 85 and 290 Hz in Figure 4) the dynamic stiffness of the damper is much greater than the dynamic stiffness of the beam at the point where it is attached, and this results in a pinning of the beam at these frequencies. The resulting structure has natural frequencies at these frequencies and hence the tuned vibration neutralizer has no beneficial effect. Indeed its effect is to create a structure that has natural frequencies which coincide with the forcing frequency. In this case, it is better to de-tune the neutralizer as suggested by Fuller *et al.* [16], so that it appears mass- or stiffness-like. The problem with a de-tuning control strategy, however, is that the kinetic energy of the host structure needs to be measured, which entails distributed measurements over the whole structure to obtain a measure of the global vibration. It would be preferable to tune the neutralizer so that it was resonant because this means that a simple control algorithm could be employed that uses only acceleration signals at the base of the neutralizer and on the neutralizer mass, as discussed by Long *et al.* [11].

To investigate the parameters that govern the large response when the neutralizer is tuned equation (11) can be arranged and the forcing term set to zero, (i.e., examine the free response of the combined system with neutralizer tuned at each frequency) to give

$$[\mathbf{I} + k_{n(tuned)}\mathbf{A}\boldsymbol{\phi}(x_n)\boldsymbol{\phi}^{\mathrm{T}}(x_n)]\mathbf{q} = 0. \tag{17}$$

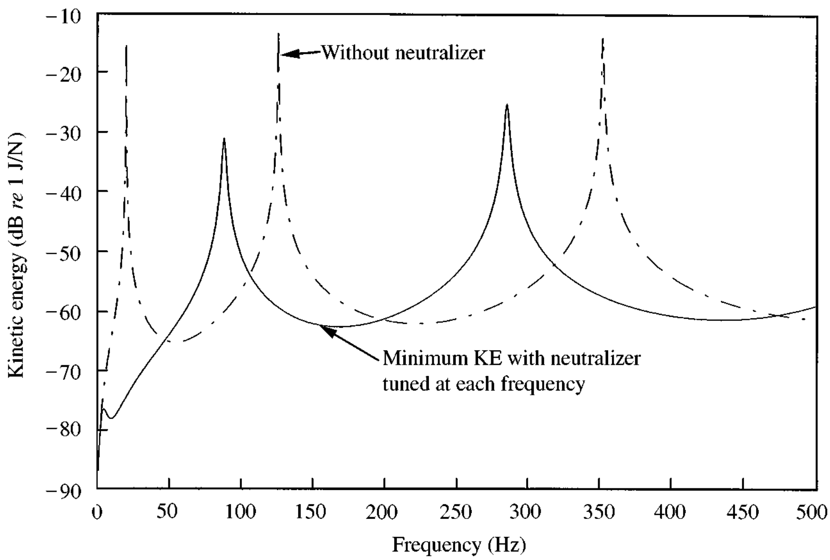

Figure 4. Kinetic energy of the cantilever beam without and with the neutralizer attached and tuned to be resonant at each frequency.

This will have natural frequencies when the determinant of the matrix is set to zero, i.e., when

$$1 + k_{n(tuned)}\alpha_{nn} = 0. \tag{18}$$

At the frequency of interest $k_{n(tuned)} \gg 0$, which means that $\alpha_{nn} \to 0$. Thus, the zeros of the point receptance, measured at the position where the neutralizer is attached, indicate the frequencies at which a tuned neutralizer cannot effect global control of the structure. Figure 5 shows the free-end point receptance of the beam pictured in Figure 2(a) without the neutralizer attached. The poles of this frequency response function are the natural frequencies of the beam without the neutralizer attached that are excited by a point force at the neutralizer position, and the zeros indicate the problematic frequencies for that neutralizer position. It is known that the zeros of receptance are a function of position on the structure [19], and hence the frequencies at which global control cannot be achieved with a tuned neutralizer are dependent upon where the neutralizer is placed on the structure. Figure 6 shows the effect of changing the neutralizer position on the natural (problematic) frequencies of the combined system. The o and * are the first and second natural frequencies of the beam, respectively, it is pinned at the neutralizer position.

The above analysis shows that provided that neutralizer is positioned correctly on the structure, it can be tuned to be a resonant device and hence global control of the structure can be achieved at a single frequency using a locally controlled tunable neutralizer. The question as to whether this control is optimal is addressed in the next section, but it is clear that provided some simple measurements are taken before fitting a tunable vibration neutralizer to ensure correct placement of the device, then a simple control system can potentially be used.

The effect of changing the position of the neutralizer can be seen in Figure 7. This figure shows the way in which the kinetic energy of the third mode of a cantilever beam, excited at position $x/L = 0{\cdot}2$, at the beams third natural frequency, by a point force, changes as a function of its position and the ratio of mass of the neutralizer to the beam, μ. It is clear

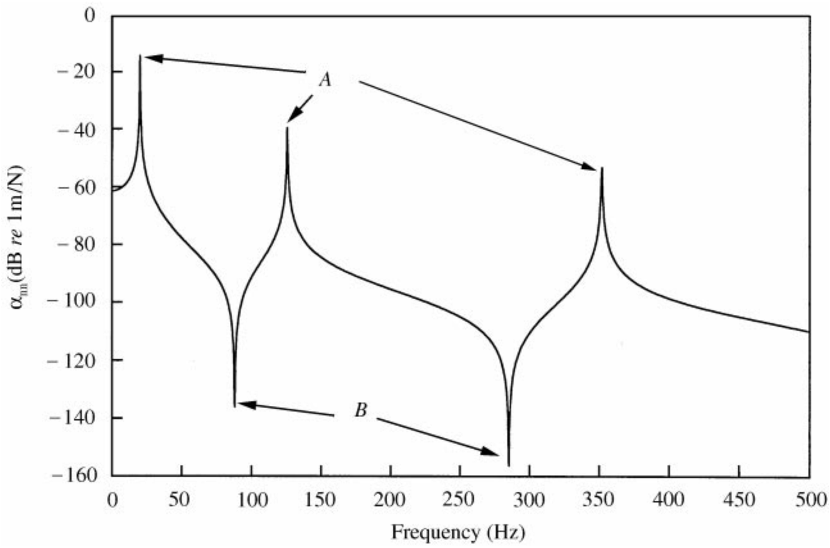

Figure 5. Point receptance of the free-end of the cantilever beam. A—natural frequencies of the beam without the neutralizer attached; B—natural frequencies of the beam pinned at the neutralizer position (free end).

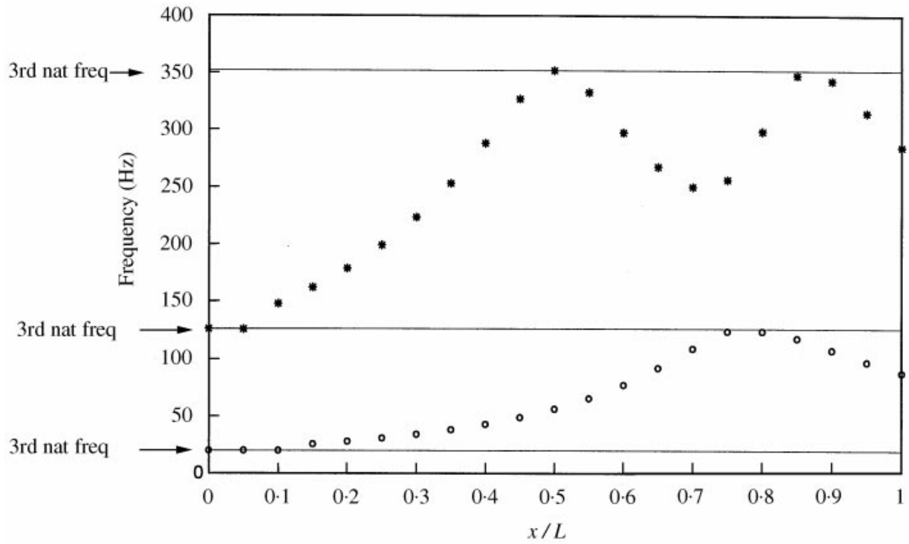

Figure 6. Graph showing how the natural frequencies of the combined system (with nodal points at the position of the neutralizer) change with the non-dimensional position of the neutralizer on the cantilever beam.

that the reduction in kinetic energy is a function of both mass ratio and position. Some general observations are

- a neutralizer placed at a nodal point on the host structure has no effect,
- a neutralizer generally becomes more effective as it is placed closer to the source,
- at some positions there appears to be a threshold mass ratio, beyond which there is no improvement in performance (for example at $x/L = 1$).

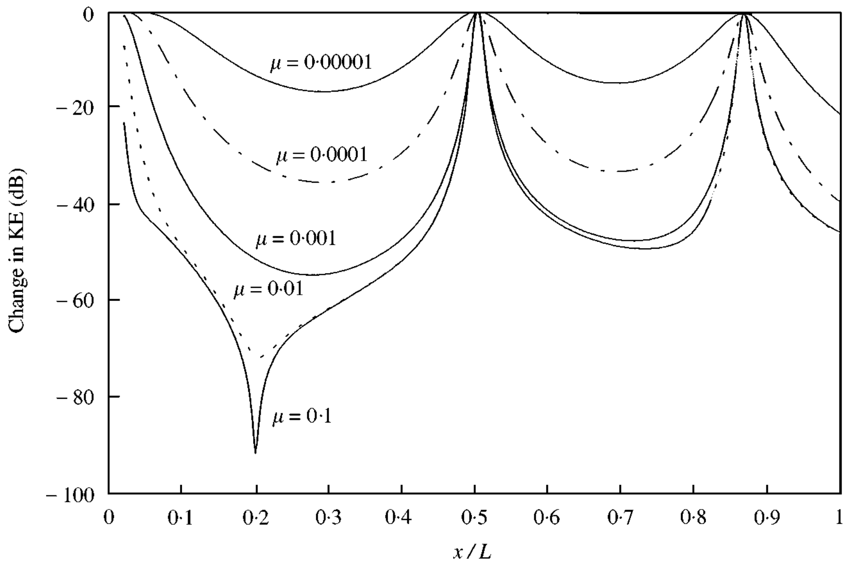

Figure 7. Change in kinetic energy of the beam as a function of non-dimensional position of the neutralizer on the cantilever beam. The beam is excited at its third natural frequency at $x/L = 0{\cdot}2$.

To investigate these effects equation (14) is examined under certain conditions. If the neutralizer is tuned to be resonant so that $|k_n| \gg 1$ and the frequency of interest is not close to a zero of the point receptance α_{nn} then

$$|k_n \alpha_{nn}| \gg 1 \tag{19}$$

and equation (14) becomes

$$\mathbf{q}_{\min} = \mathbf{A}\left[\boldsymbol{\phi}(x_f) - \boldsymbol{\phi}(x_n)\frac{\alpha_{nf}}{\alpha_{nn}}\right] f \tag{20}$$

which can be substituted into equation (10) to determine the minimum kinetic energy of the structure. Inspection of equation (20) shows that provided the condition given in equation (19) holds, then surprisingly the kinetic energy of the host structure is independent of the dynamic stiffness of the tuned neutralizer. If the neutralizer is placed at the excitation position then the subscript f becomes n and $\mathbf{q}_{min} = \mathbf{0}$, which means that the kinetic energy of the beam can, in principle, be set to zero. It is possible to determine an expression for the optimum dynamic stiffness of the neutralizer provided attention is restricted to a frequency close to the mth natural frequency of the original structure. In this case the response is governed by the amplitude of the mth mode which can be determined from equation (14) and is given by

$$q_m = \frac{A_m \phi_m(x_f) f}{1 + k_n A_m \phi_m^2(x_n)}. \tag{21}$$

Normalizing this to the modal amplitude of the structure without the neutralizer attached, and substituting for A_m with $\omega = \omega_m$, and the tuned dynamic stiffness $k_{n(tuned)}$,

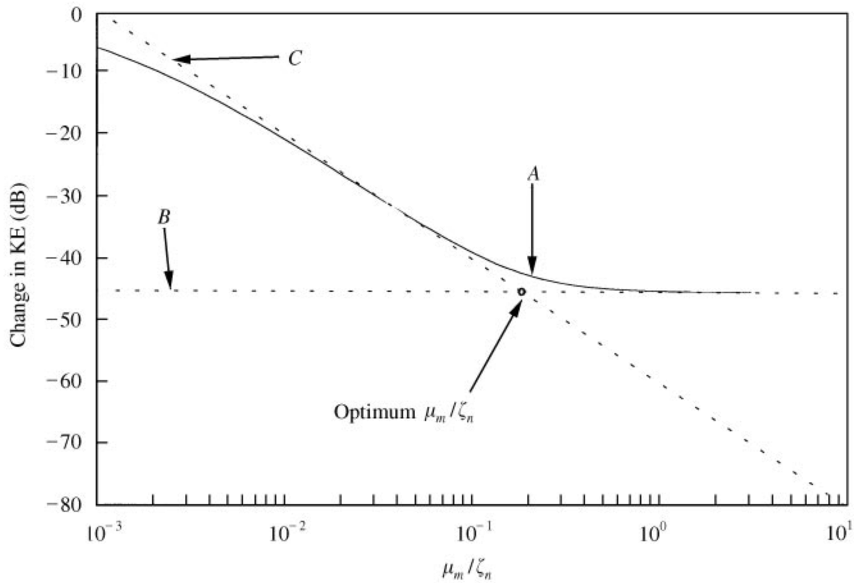

Figure 8. Change in the kinetic energy of the beam at the third natural frequency as a function of mass and damping ratios: A—actual change in the kinetic energy; B and C are asymptotes for high and low μ_m/ζ_n ratios respectively.

gives

$$q_{m(norm)} = \frac{1}{1 + \mu_m \phi_m^2(x_n)/4\zeta_m\zeta_n}. \tag{22}$$

It is clear that this mode reduces in amplitude as the mass ratio increases, and the damping in the neutralizer and the structure decrease. The positional dependence is also apparent; the neutralizer is most effective if it positioned on an antinode, and is ineffective if it is placed on a node. Equating the kinetic energy calculated using equations (21) and (10) with the kinetic energy calculated using equations (20) and (10) with $\omega = \omega_m$ and $k_n = k_{n(tuned)}$, the optimum tuned dynamic stiffness of the neutralizer can be determined:

$$\left(\frac{1}{1 + \mu_m \phi_m^2(x_n)/4\zeta_m\zeta_n}\right)^2 = \left(\frac{2\zeta_m\omega_m^2}{\phi_m(x_f)}\right)^2 \mathbf{b}^H\mathbf{b}, \tag{23}$$

where $\mathbf{b} = m\mathbf{q}_{min}$, and μ_m is the ratio of the mass of the neutralizer to the mass of the mth mode of the structure. Assuming that $\mu_m\phi_m^2(x_n) \gg 4\zeta_m\zeta_n$ equation (23) can be rearranged to give

$$\left.\frac{\mu_m}{\zeta_n}\right|_{\mathrm{opt}} = \frac{2|\phi_m(x_f)|}{\phi_m^2(x_n)\,\omega_m^2\sqrt{\mathbf{b}^H\mathbf{b}}}. \tag{24}$$

The change in the kinetic energy of the cantilever beam pictured in Figure 2(b), excited at its third natural frequency is shown in Figure 8 as a function of the mass ratio divided by the neutralizer's damping ratio. The beam is excited at $x/L = 0{\cdot}2$ and the neutralizer is positioned at the free-end and has a mass ratio of 0·1; the beam and neutralizer damping ratios are set at 0·001. The actual change in the kinetic energy was calculated using equation (10) and equation (14), and is labelled A; the normalized minimum kinetic energy was calculated using equation (10) and equation (20) normalized by the kinetic energy of the third mode, and is labelled B; the sloping line was calculated using equations (10) and (22) with the 1 in the denominator neglected, and is labelled C. The optimum ratio μ_m/ζ_n was calculated using equation (24). It can be seen that this is quite a good approximation, and is thus considered useful in the design of a vibration neutralizer for a particular structure. It is interesting to note that as the neutralizer is moved closer to the source the maximum reduction in the kinetic energy increases. However, to achieve the maximum reduction, the optimum μ_m/ζ_n ratio increases, which means that either the mass of the neutralizer has to increase or the damping in the neutralizer has to decrease.

It is tempting to reduce the damping in the neutralizer, because adding mass in a vibration control device is usually not encouraged. However, it is well known that the separation between the peaks on either side of the operating frequency is governed by the mass ratio [2], and clearly it is desirable to have a "reasonable" peak separation so that the system is reasonably robust to rapid changes in excitation frequency as discussed by Brennan [4]. To derive an expression for this peak separation, k_n from equation (6) is substituted into equation (21) for, with damping in the structure and the neutralizer both set to zero. Setting $\omega_n = \omega_m$, gives

$$q_m = \frac{(1 - \Omega^2) f}{m(1 - \Omega^2)^2 - \phi_m^2(x_n)\mu_m \Omega^2)}, \tag{25}$$

where $\Omega = \omega/\omega_m$. The frequencies at which the peaks occur can be determined by setting the denominator of equation (25) to zero. This results in a quadratic equation with the solutions:

$$\Omega_{1,2}^2 = 1 + \frac{\phi_m^2(x_n)\mu_m}{2} \pm \sqrt{\left(1 + \frac{\phi_m^2(x_n)\mu_m}{2}\right)^2 - 1}\,. \tag{26}$$

Forming the difference $\Delta\Omega = \Omega_1 - \Omega_2$ and substituting the two roots from equation (26), gives, after some algebraic manipulation

$$\Delta\Omega = |\phi_m(x_n)|\mu_m^{1/2}. \tag{27}$$

Thus, by using equations (24) and (27) the optimum neutralizer mass and damping ratio can be determined for global vibration control once a tolerable peak separation has been defined, and this will be dependent upon the control delay in the neutralizer as discussed by Brennan [4].

4. COMPARISON BETWEEN ACTIVE CONTROL AND CONTROL USING A TUNABLE VIBRATION NEUTRALIZER

In this section global control of vibration is compared using the following: (a) a passive device with no restrictions on the passive elements (optimal passive control), (b) a tunable vibration neutralizer, and (c) fully active control, utilizing the theory described by Nelson and Elliott [20] for the active control of sound. The problem is formulated with the attached dynamic stiffness of the control device being the control variable. With strategy (a), the imaginary part of the dynamic stiffness is constrained to be positive, so no energy is supplied by the control device, with strategy (b) the imaginary part is constrained to be

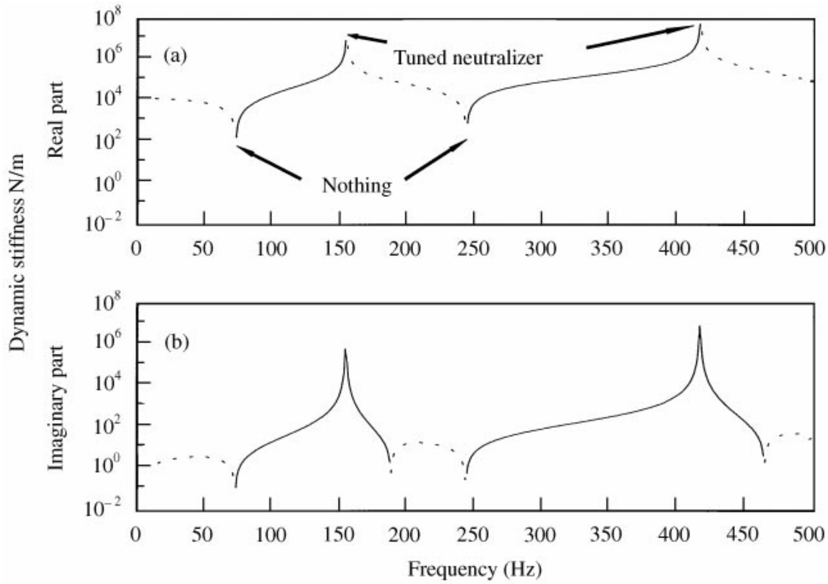

Figure 9. Optimal dynamic stiffness of the neutralizer as a function of frequency, (a) is the real part: - - - mass; —— stiffness and (b) is the imaginary part: - - - supply energy; —— absorb energy. Solid line—positive; dashed line—negative.

positive and the real part is set to infinity, and with strategy (c), there are no constraints on the dynamic stiffness as it can supply and absorb energy.

Equation (14) can be written as

$$\mathbf{q} = \mathbf{d} + \mathbf{C}\left(\frac{k_n}{1 + k_n \alpha_{nn}}\right), \tag{28}$$

where

$$\mathbf{d} = \mathbf{A}\boldsymbol{\phi}(x_f)f \quad \text{and} \quad \mathbf{C} = -\mathbf{A}\boldsymbol{\phi}(x_n)\alpha_{nf}f. \tag{29a, b}$$

When equation (28) is substituted into equation (10) to give the kinetic energy, the resulting equation is of Hermitian quadratic form, which has a minimum when [20]

$$k_{n(opt)} = \frac{-[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H\mathbf{d}}{1 + \alpha_{nn}[\mathbf{C}^H\mathbf{C}]^{-1}\mathbf{C}^H\mathbf{d}} \tag{30}$$

and the minimum kinetic energy can be found by setting k_n to $k_{n(opt)}$ in equation (28), which in turn is substituted into equation (10). The optimum dynamic stiffness has real and imaginary parts which are plotted in Figures 9(a) and (b), respectively, for the cantilever beam used in the earlier simulations with the dynamic stiffness placed at the free-end of the beam. It can be seen that both the real and imaginary parts are both positive and negative depending upon frequency. The interpretation of these graphs is as follows. The real part of the dynamic stiffness is related to the reactive passive elements of mass and stiffness. A positive real part is means that the dynamic stiffness should be stiffness-like, and conversely a negative real part corresponds to a mass-like dynamic stiffness, and this is shown in Figure 9(a). If the real part of the optimum dynamic stiffness is infinite then this

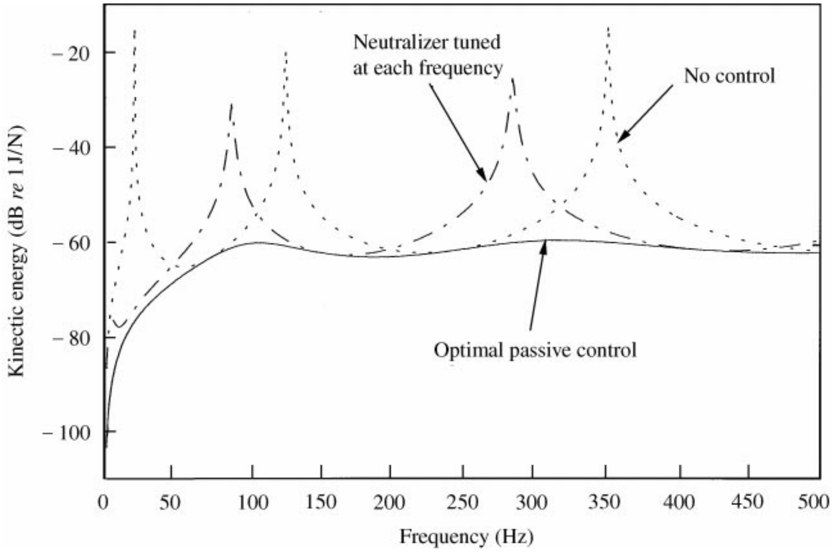

Figure 10. Comparison of the kinetic energy of the cantilever beam between optimal passive control and control using a tuned vibration neutralizer tuned at each frequency.

corresponds to a vibration neutralizer, and if it is zero then this corresponds to no mass or stiffness. The imaginary part of the dynamic stiffness corresponds to damping. If it is positive then this means that the device should absorb energy, and if it is negative then it should supply energy to the structure. It can be seen from Figure 9(b) that at some frequencies then energy needs to be supplied to the beam, but at other frequencies the device should absorb energy. However, it should be noted that the imaginary parts of the dynamic stiffness are several orders of magnitude less than the real part, and when it is set to zero for the example considered in this paper, it makes little difference to the resulting kinetic energy of the beam. Thus, it can generally be seen that if an active control system is used to control global vibration, then at most frequencies it effectively has to synthesize an attached mass or a stiffness. At certain frequencies it has to synthesize a tuned vibration neutralizer or nothing at all.

From these simulations it is clear why a de-tuning control strategy has been used to control global vibration. However, this is not a simple strategy to implement in practice, as discussed above, and so it is worthwhile to see how effective a tuned vibration neutralizer is compared with optimal control, but with the imaginary part of the dynamic stiffness constrained to be positive. Figure 10 shows the kinetic energy of the cantilever beam calculated at each frequency with the two control strategies implemented; the kinetic energy before control is also shown. It can be seen that the constrained optimal control strategy reduces the kinetic energy at most frequencies, and never makes the situation worse. The best control is achieved around the original resonance frequencies of the beam, and there are some frequencies when no control is possible. It is also clear that although the tuned vibration neutralizer is effective at some frequencies, it makes global vibration worse at other frequencies, as discussed previously. Dividing the kinetic energy of the beam with an optimally controlled device attached by the kinetic energy of the beam with an optimally controlled device attached by the kinetic energy of the beam with a tuned (at each single frequency) neutralizer attached, the frequency range over which the tuned neutralizer is effective can be clearly identified. This is shown in Figure 11. There are large frequency

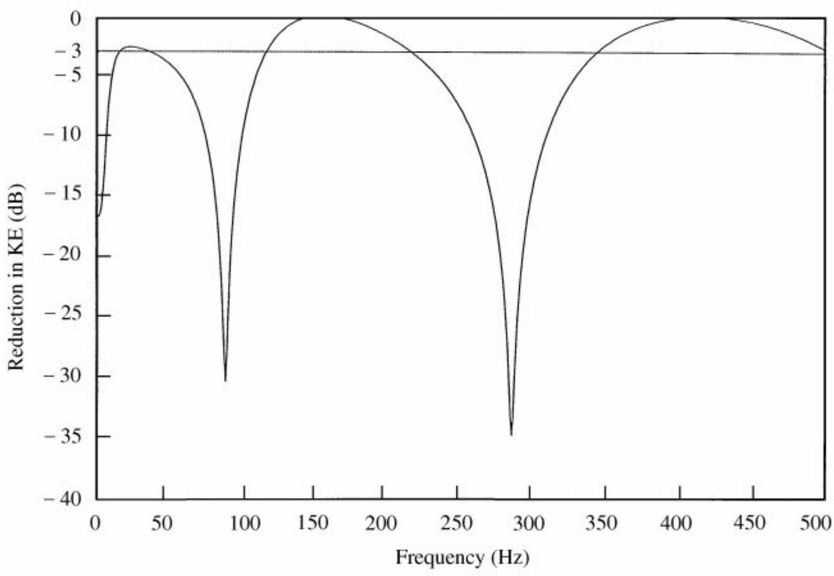

Figure 11. Ratio of the kinetic energy of the beam with optimal passive control to the kinetic energy of the beam controlled with a neutralizer, tuned at each frequency.

ranges, for example from about 120–220 and 340–500 Hz where the tunable vibration neutralizer's effectiveness is within 3 dB of the optimal passive control. These frequency ranges can be adjusted by careful choice of neutralizer position as discussed in section 2. Thus, it is possible to use a resonant vibration neutralizer tuned using a local control strategy to control the global vibration of a structure. This control strategy, although sub-optimal, gives results that are very similar to a de-tuning control strategy, which requires a more sophisticated control system.

5. CONCLUSIONS

In this paper the use of a tunable vibration neutralizer has been investigated for the control of global vibration of a structure. A general mathematical model has been developed for a structure with many vibration neutralizers attached, but to gain some physical insight into the control mechanisms a beam with a single device attached has been studied. With local vibration control, where the aim is for the vibration neutralizer to pin the structure at the point of attachment, the dynamic stiffness of the neutralizer is required to be as large as possible. However, with global control the required dynamic stiffness of the neutralizer has an optimum (threshold) value, beyond which any increase does not result in improved performance, and an expression for this optimum value has been derived. A tuned vibration neutralizer can, at some frequencies, result in an increase rather than decrease in global vibration. These frequencies are the natural frequencies of the host structure when it is pinned at the neutralizer attachment point. If these natural frequencies coincide with a frequency of interest then they can be shifted to other frequencies by changing the position of the neutralizer. It has been shown that by correctly positioning the neutralizer it can be tuned to be a resonant device and effect global control of the structure, that offers a performance within 3 dB of that achievable with an optimal passive control device.

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APPENDIX A

In this Appendix it is shown that

$$[\mathbf{I} + k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)]^{-1} = \left[\mathbf{I} - \frac{k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)}{1 + k_n \sum_{m=1}^{M} A_m \phi_m^2(x_n)} \right]. \tag{A1}$$

When there is only a single neutralizer, $\boldsymbol{\phi}$ is an M-length vector, $\mathbf{A}$ is an $(M \times M)$ diagonal matrix and k_n is a scalar. Thus, the matrix $[k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)]$ has a rank of 1 and has eigenvalues of $\lambda_1 = 0$ and $\lambda_2 = \mathrm{Trace}\,[k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)] = k_a \sum_{m=1}^{M} A_m \phi_m^2(x_n)$. Thus, the eigenvalues of $[\mathbf{I} + k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)]$ are $\lambda_1 = 1$ and $\lambda_2 = 1 + k_a \sum_{m=1}^{M} A_m \phi_m^2(x_n)$. Letting

$$\mathbf{Z} = [\mathbf{I} + k_n \mathbf{A} \boldsymbol{\phi}(x_n) \boldsymbol{\phi}^{\mathrm{T}}(x_n)], \tag{A2}$$

the Cayley–Hamilton theorem [21] can be used to write

$$\mathbf{Z}^p + c_1 \mathbf{Z}^{p-1} + \cdots + c_{p-1} \mathbf{Z} + c_p \mathbf{I} = 0, \tag{A3}$$

where the c's are the coefficient of the characteristic equation of the matrix $\mathbf{Z}$, which is given by

$$(\lambda_1 - 1)\left(\lambda_2 - \left(1 + k_a \sum_{m=1}^{M} A_m \phi_m^2(x_n)\right)\right) = 0. \tag{A4}$$

The resulting coefficients of the characteristic equation are

$$c_1 = -2 - k_a \sum_{m=1}^{M} A_m \phi_m^2(x_n), \quad c_2 = 1 + k_a \sum_{m=1}^{M} A_m \phi_m^2(x_n). \tag{A5a,b}$$

Rearranging equation (A3), multiplying by $\mathbf{Z}^{-1}$ and setting $p = 2$, gives

$$\mathbf{Z}^{-1} = \frac{(-c_1 \mathbf{I} - \mathbf{Z})}{c_2}, \tag{A6}$$

Substituting for $\mathbf{Z}$ from equation (A2) and c_1 and c_2 from Equations (A5a,b) into equation (A6) gives equation (A1) as required.